REAL-time monitoring and mitigation of nonlinear effects in optical **NET**works (REAL-NET)

D1.2 Expermintal validation of the simplified SSFM and volterra series in real time transmission systems and their comparison

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Abstract

In this deliverable, we provide a review of the optical communication systems. A mathematical description for the modulator and the demodulator is presented . We investigate the nonlinear Schrödinger's equation (NLSE) that is used to model the linear and the nonlinear effects introduced by the fiber channel. We present a numerical approach called the split step Fourier method (SSFM) to solve the NLSE and compute the output of the channel.We review the Volterra series as another solution to the NLSE equation. We introduce the neural networks from machine learning. The compensation of nonlinear effects based on digital back propagation (DBP),Volterra series and neural networks will be reviewed . Finally simulation results to compare the DBP and the neural network will be shown.

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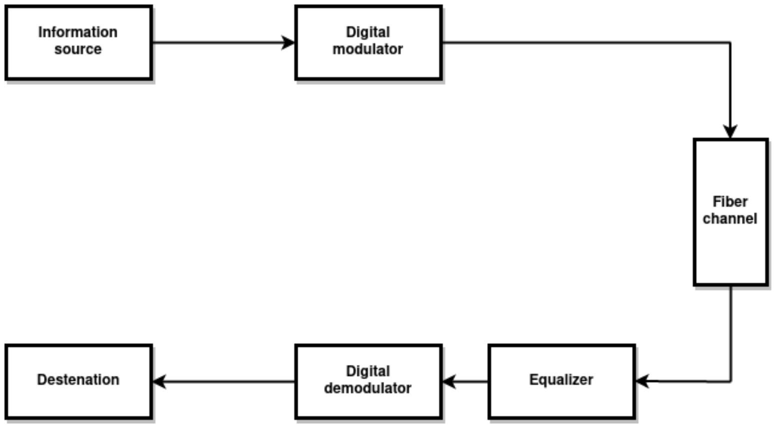

Fig. 1. Basic components of the digital communication system

1 Introduction

Pulse propagation model is essential to investigate various effects in nonlinear optical fiber [1]. One of the efficient techniques that is used for implementing the nonlinear Schrödinger's equation (NLSE) which describes the pulse propagation phenomenon, is the split split step Fourier method (SSFM). The mitigation of the deterministic nonlinear effects is performed by inverting the SSFM, the inversion process is known as the digital back propagation (DBP) [2].It has been noted that DBP can require high computational resources. [3].

Compensation of the nonlinear distortions using neural networks has attracted a lot of attention recently [4], [5]. However, the design of the neural networks can be difficult, and there are no specific guidelines on how to set the hyperparameters for a given problem.

In section (2), the components of the fiber communication system are demonstrated. The mathematical description of the modulator and the demodulator is then provided. The channel model for a single polarization mode fiber is presented, followed by equalization using DBP and Volterra series approach. In section (3), a basic introduction to the neural networks is provided. Simulation results for the neural network are presented and compered with the DBP. In section (4) a summary is presented.

2 Digital fiber-optic communication system

The basic components of a digital communication system are presented in this section. The main goal of a digital communication system is the transmission of digital information from a source and reproducing the transmitted signal at the destination.

The functional diagram of the digital communication system is shown in 1. The output of the information source is assumed to be in the digital form. The binary sequence is then passed to the digital modulator, which serves as the interface to the channel. The digital modulator in the case of memory less modulation, partitions the data stream into binary sequences and then maps them into signal waveforms that match the channel. The channel is the physical medium, through which the signal propagates. The essential feature for the physical media is that the transmitted signal is corrupted in a random manner by variety of mechanisms. The corrupted signal at the receiving end of the channel is then passed to the equalizer, which mitigates the corruption mechanisms introduced by the channel. The digital demodulator reproduces the transmitted sequence.

2.1 Modulator

The purpose of the digital modulator is to produce a signal that matches the characteristics of the channel and be able to a certain level to resist the impairments caused by the channel. As presented in Fig. 2 the digital modulator can be decomposed into three parts. The mapper is responsible for mapping the digital data stream into discrete time symbols. The mapping could be memoryless or with memory resulting in memoryless modulation schemes or modulation schemes with memory. The waveshaper module converts the discrete time symbols into baseband wave forms that is continuous in time and amplitude by multiplying the mapper generated symbols with

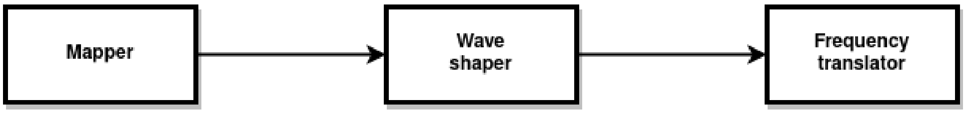

Fig. 2. Digital modulator

continuous time functions. The choice of the function determines the spectral characteristics of the transmitted signal, such as bandwidth. The frequency translator shifts the frequency of the baseband signal into the bandpass region. In memoryless modulation, the data stream is segmented into sequences of length k and mapped into a waveform. Each segment is independent from the previous one and each waveform is send every T_s seconds or every symbol interval, this means that in each second R_s symbols are transmitted

$$R_s = \frac{1}{T_s}, \tag{1}$$

Where R_s is called the signaling rate. Each signal carries k bits of information, thus

$$T_b = \frac{T_s}{k}, \tag{2}$$

where T_b is called the bit interval. The bit rate is defined as

$$R_b = kR_s; \tag{3}$$

The transmitted signal $\bar{Q}(t)$ can be written in terms of its low pass equivalent $Q(t)$

$$\bar{Q}(t) = \Re[Q(t)\exp(j2\pi f_c t)], \tag{4}$$

where f_c is the carrier frequency. The complex envelope of the transmitted signal can be written as

$$Q(t) = \sum_{i=-M/2}^{M/2-1} s_i P(t - iT_s), \tag{5}$$

where $P(t)$ is the pulse shape supported on T_s seconds and s_i is the transmitted symbol at time instance i. We assume that (s_i) is an iid sequence with $E|s_i|^2 = P$ and that the pulse shape has unit power, *i.e.*, $(1/T_s)\int_{-\infty}^{\infty}|P(t)|^2 dt = 1$.

The average power of the signal is

$$P_a = \frac{1}{MT_s}\sum_{i=-M/2}^{M/2-1} E|s_i|^2 \int_{-\infty}^{\infty} |P(t)|^2(t)dt \tag{6}$$

$$= P. \tag{7}$$

2.2 Fiber channel model

In fiber communication, the generated electrical signal is converted into light wave either by a light emitting diode or a laser. The intensity of the light is varied with the message signal. The transmitted light wave is amplified periodically by amplifiers along the transmission path to mitigate the attenuation of the signal.

One of the distortions introduced by the fiber is the loss. The transmitted wave often suffers form power loss due to several factors such as material absorption and the phenomenon of Rayleigh scattering. Since these phenomena depend on the wavelength of the wave, fiber losses are wavelength dependent. Denote P_0 as the lunched power at the input of the fiber with length L. The transmitted power can be given by:

$$P_T = P_0 \exp(-\alpha L), \tag{8}$$

where α is the attenuation constant.

Another important distortion limiting the data rate of fiber optic communication systems is the chromatic dispersion. Different signal frequencies (colors) travel at different speeds causing the spectral components of the light wave to arrive at different times at the output, thereby broadening the transmitted pulse. One factor that contributes to this phenomenon is the dependency of the refractive index on the wavelength, which is approximated by the Sellmeier equation:

$$n^2(w) = 1 + \sum_{j=1}^{m} \frac{B_j w_j^2}{w_j^2 - w^2}, \tag{9}$$

where w_j is the resonance frequency and B_j is the strength of the jth resonance.

The effect of the chromatic dispersion can be seen by expanding the mode propagation constant β around the frequency at which the pulse spectrum is centered w_0

$$\beta(w) = \beta_0 + \beta_1(w - w_0) + \frac{1}{2}\beta_2(w - w_0)^2 + \cdots, \tag{10}$$

where

$$\beta_m = \left(\frac{d^m \beta}{dw^m}\right)_{w=w_0}.$$

In optical fiber the refractive index also depends on the intensity of the electromagnetic wave that propagates through, the refractive index can be written in the form:

$$n(w, I) = n(w) + n_2 I, \tag{11}$$

where $n(w)$ is defined in 9, n_2 is the nonlinear refractive index and I is the intensity of the wave. One of effects induced by this nonlinear phenomenon is the self induced phase modulation, where a nonlinear phase shift is produced in the travelling wave.

A travelling wave in the optical medium also suffers from noise. Noise is defined as the deviation from an ideal signal, it is usually associated with random processes. By definition it corrupts the information content and fidelity of the signal. The main types of noise can be classified into various categories such as, thermal noise that is related to the motion of the electrons and shot noise, which is produced by the amplifiers in the system. In our analysis we assume that the noise effecting the signal is white over the bandwidth of the travelling signal.

The pulse propagation in a single polarization mode fiber is modeled by the nonlinear Schrödinger equation

$$\frac{\partial Q(z,t)}{\partial z} = -\frac{\alpha}{2} Q(z,t) - j \sum_{k=1}^{\infty} \frac{j^k}{k!} \beta_k \frac{\partial^k Q(z,t)}{\partial t^k} - j\gamma |Q(z,t)|^2 Q(z,t) + N(z,t), \tag{12}$$

where $j = \sqrt{-1}$. $|.|$ represents the magnitude. $N(t)$ is the channel noise. $Q_l(z,t)$ is the low pass representation of the propagating wave at distance z. α is the attenuation constant. $\beta_k = \frac{\partial^k \beta}{\partial t^k}|_{w=w_0}$ is the kth dispersion coefficient. γ is the nonlinearity parameter. In most scenarios (12) can not be solved analytically and requires a numerical solution.

2.3 Split step Fourier method

In SSFM, the fiber channel is divided into small segments Δz of length $\frac{z}{n}$, where n is an integer chosen to be large. In each segment its assumed that the linear and the nonlinear effects act independently from each other and can be solved separately. the linear part of the NLSE is written as

$$\frac{\partial Q_l(z,t)}{\partial z} = -\frac{\alpha}{2} Q_l(z,t) - j \sum_{k=1}^{\infty} \frac{j^k}{k!} \beta_k \frac{\partial^k Q_l(z,t)}{\partial t^k}. \tag{13}$$

Define $Q_l(z,w)$ as the Fourier transform of $Q_l(z,t)$

$$Q_l(\Delta z, w) = \int_{-\infty}^{\infty} Q_l(z,t) \exp(jwt) dt. \tag{14}$$

In frequency domain (13) is presented as

$$\frac{\partial Q(z,w)}{\partial z} = -\frac{\alpha}{2} Q(z,w) - jQ(z,w) \sum_{k=1}^{\infty} \frac{j^k}{k!} \beta_k (w)^k. \tag{15}$$

The differential equation (15) can be solved

$$Q_l(\Delta z, w) = Q_l(0, w) \exp\left(-\frac{\alpha}{2}\Delta z - j\Delta z \sum_{k=1}^{\infty} \frac{j^k}{k!}\beta_k(w)^k\right). \tag{16}$$

The nonlinear part of the NLSE is written as

$$\frac{\partial Q_{nl}(z,t)}{\partial z} = -j\gamma |Q_{nl}(z,t)|^2 Q_{nl}(z,t). \tag{17}$$

Equation (17) is solved

$$Q_{nl}(\Delta z, t) = Q_{nl}(0,t)\exp(-j\gamma\Delta z|Q_l(\Delta z, t)|^2). \tag{18}$$

In each segment of the fiber channel, the linear effect is add first and the output of the linear system is considered as the input of the nonlinear system.

The approach presented above can be modified for each segment to add a linear step for $\frac{\Delta z}{2}$, then the nonlinear step and finally add the remaining half of the linear step to the signal, this is called symmetric SSFM.

The DBP algorithm is done by inverting the the fiber parameters, then inputting the received signal into the inverted SSFM model.

2.4 Volterra series approach

The Volterra series is a generalization of the impulse response for linear systems, describing the nonlinear action of the channel. For fiber with one polarization, the Volterra series expresses the output signal in terms of the input signal as follows [6]:

$$\begin{aligned} Q(z,t) &= \int_{-\infty}^{\infty} h^{(1)}(\tau) Q(0, t-\tau) d\tau \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(3)}(\tau_1,\tau_2,\tau_3) Q(0,t-\tau_1) Q^*(0,t-\tau_2) Q(0,t-\tau_3) d\tau_1 d\tau_2 d\tau_3, \end{aligned} \tag{19}$$

where $h^n(.)$ is the nth order Volterra kernel, $*$ is the conjugate operation and we ignored the higher order terms beyond cubic.

The Volterra series is often implemented in the frequency domain for nonlinearity compensation in the optical fiber communication. In the frequency domain (19) is expressed as [6]

$$\begin{aligned} Q(z,w) &= h^{(1)}(w) Q(0,w) \\ &+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h^{(3)}(w, w_1, w_2, w_3) Q(0,w_1) Q^*(0,w_2) Q(0,w_3) \delta_{123w} dw_1 dw_2 dw_3. \end{aligned} \tag{20}$$

where

$$\delta_{123w} := \delta(w_1 - w_2 + w_3 + w) \tag{21}$$

where $\delta(x)$ is Dirac Delta function.

The first and the third Volterra kernels are obtained in [6]

$$h^{(1)}(w) = \exp(jw^2 z), \tag{22}$$

$$h^{(3)}(w, w_1, w_2, w_3) = -jze^{-jz\frac{1}{2}(w^2 - w_1^2 + w_2^2 - w_3^2)} \text{sinc}\left(\frac{1}{2}z(w^2 - w_1^2 + w_2^2 - w_3^2)\right), \tag{23}$$

where sinc(x) $= \frac{\sin x}{x}$.

The Volterra equalizer takes the inverse of the channel by inverting the Volterra kernels. The performance of DBP and Volterra series for nonlinearity compensation are compared in [7,8]. In most cases, DBP requires more FFT operations compared to the Volterra series approach making it more complex [7,8].

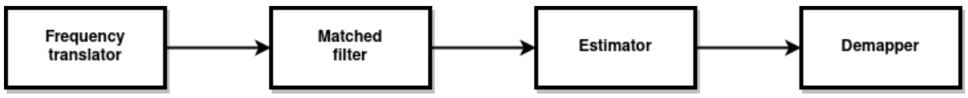

Fig. 3. Digital demodulator

2.5 Demodulator

At the receiving side, the equalized signal is converted into the baseband form by the frequency translator. The complex envelope of the signal is then projected into the Cartesian coordinate system. The estimator takes the projected signal and produces an estimate of the transmitted symbol by comparing the received symbols with the all possible symbols that is in the constellation. The demapper maps the estimated symbols into there bit representation.

The received symbol at the ith signaling interval is obtained by:

$$s_i = \int_{-\infty}^{\infty} Q(t)P(t - iT_s)dt \tag{24}$$

$$= \hat{s}_i + \zeta, \tag{25}$$

where ζ is the noise term. The optimal symbol decision rule is

$$\bar{s} = \arg\min_{s_i} |s_i - \hat{s}_i|. \tag{26}$$

3 Introduction to neural networks

3.1 Review

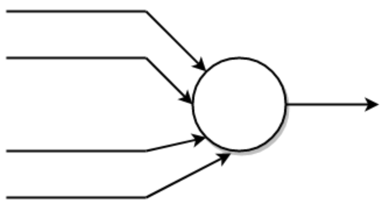

Fig. 4. Neuron

Before providing a definition for the neural networks, first we investigate the neuron which serves as the building block of this type of networks. The neuron takes several inputs and produces a single output by employing the following equation:

$$y = f_{act}\left(\sum_{i=1}^{N} w_i x_i + b\right), \tag{27}$$

where $x_i \in R$ for $i = 1, 2, \cdots, N$ is the input of the neuron, $w_i \in R$ is the corresponding weight of the input x_i, $b \in R$ is the bias and f_{act} is a function called the activation function of the neuron.

The neural network is defined as an interconnected group of neurons. Neurons are aggregated into layers. Different layers may perform different transformations on their inputs. The input travels from the first layer (the input layer) to the last layer (the output layer). The input layer does not perform any function on the input, it just maps the input into the next layer. The layers between the input and the output layer are called the hidden layers and they perform transformations on their inputs. The last layer called the output layer that produces the output of the system.

By setting the wights, biases and the activation functions for the neurons of the network a certain output will be generated. What makes the neural network unique is that with proper training the neural network has the ability to learn the mapping function between the input and output by varying its own weights and biases.

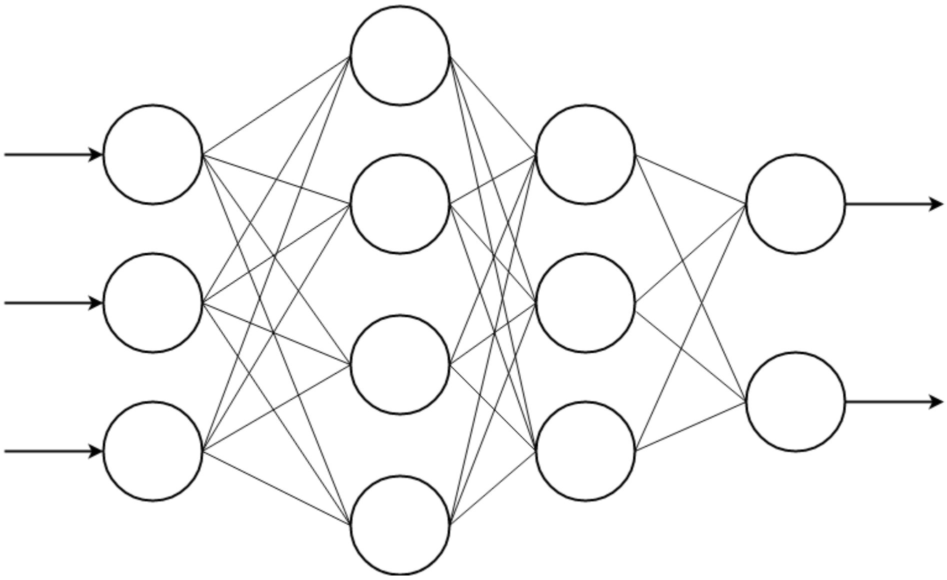

Fig. 5. Multi neurons network

parameter	value
fiber length	2000 km
α	0
β_2	-21.67*10^{-27} s^2/m
γ	1.27*10^{-3} Watt^{-1}/m

TABLE 1. Fiber parameters

In order for the neural network to perform the learning task, the neural network must have the access to the input data $\mathbf{X} = (\mathbf{x^{(1)}}, \mathbf{x^{(2)}}, \cdots, \mathbf{x^{(m)}})$,$\mathbf{x^{(i)}} \in R^N$, the true output $\mathbf{y} = (\mathbf{y^{(1)}}, \mathbf{y^{(2)}}, \cdots, \mathbf{y^{(m)}})$,$\mathbf{y^{(i)}} \in R^N$. It also must have a measures of success described by a loss function.

The real distribution of the input and the output are not known to the neural network, thus the approach is to minimize the empirical loss function:

$$L_S = \frac{1}{m}\sum_{i=1}^{m} l(h_S(\mathbf{x^{(i)}}), \mathbf{y^{(i)}}), \tag{28}$$

where h_S is the predictor function of the neural network, $l(x, y)$ is some defined loss function.

Minimizing the empirical loss function is called empirical risk minimization and its done in the neural network by back propagation, in which the empirical loss for the predictor is propagated back from the output layer to the input layer and the wights and biases of the neural networks are updated using gradient descent method.

In the training of the neural network even if the training error (empirical error) is small that does not indicate the neural network is learning. It means that the neural network is optimizing well and does not reveal any information about the behavior for unseen data. Large neural networks can exhibit a some difference between the training error and the generalization error.

3.2 Simulation results

In this section, we present some simulation results showing the nonlinear effects of the fiber channel, the equalization using the DBP algorithm and the neural network. In figure (6) we show the equalization for one symbol and multi symbols transmission.

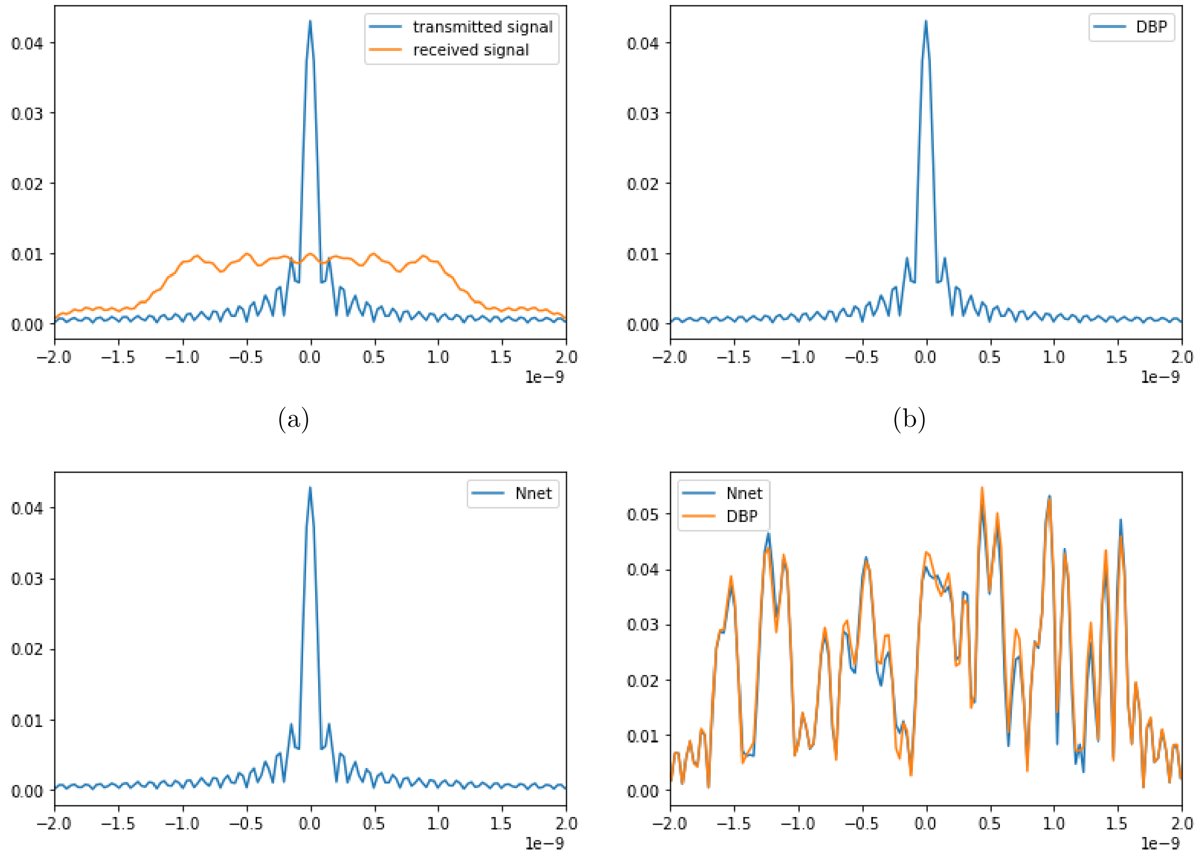

Fig. 6. (a) Transmitted and received signals with one symbol; (b) equalization using DBP; (c) equalization using the neural network; (d) equalization for multiple symbols. Horizontal and vertical axes are t in second and $|Q(z,t)|$ in $\sqrt{\text{Watt}}$ respectively.

4 Conclusions

In this deliverable, we reviewed the optical communication system. We started by presenting the components of the digital communication system such as the modulator, demodulator, equalizer and the fiber channel. The pulse propagation model that includes the linear and nonlinear effects of the fiber channel was investigated. A numerical solution SSFM was utilized to solve the NLSE and compute the channel output. The Volterra series approach was also used to solve the NLSE. DBP and Volterra series based equalization algorithms were presented and utilized in the equalization of the received signal. An introduction to the neural networks was reviewed. Simulation results for the channel and equalization using the DBP algorithm and the neural network were shown.

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